# Hard diffractive electroproduction of heavy flavor vector mesons in QCD

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### Abstract

We outline $QCD$ predictions for the diffractive electroproduction of heavy flavor vector mesons and compare them with available data.

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# 1 Diffractive electroproduction of heavy flavor vector mesons in QCD

It has been understood recently that at sufficiently large $Q^2$ diffractive electroproduction of vector mesons is calculable in $QCD$ [1]. The necessary conditions are: i) $\frac{1}{2m_N x}$ should be much larger than the radius of the target so that the photon transforms predominantly into a quark component well before the target; ii) high $Q^2$ is a necessary but not sufficient condition to guarantee small interquark distances in the quark component of the wave function of the projectile photon, and thus enables the applicability of the assumption of asymptotic freedom; iii) the initial photon should be longitudinally polarized to ensure the smallness of the produced quark configuration when it arrives at the target. Another potentially fruitful handle to guarantee the smallness of the interquark distances in the wave function of the photon is the large mass of the heavy quark ($m_q \to \infty$). M.Ryskin [2] suggested that the large mass of the $c$ quark is a sufficient condition for the applicability of $pQCD$ for the production of $c$ quarks in the with a proton target and to ensure the applicability of the charmonium model to describe the light–cone wave function of the produced $J/\psi$ meson.

When the momentum transferred to the target- $t$ tends to zero, the cross section for the hard diffractive electroproduction of $J/\Psi$ through the interaction of a longitudinally polarized photon with a target proton at sufficiently large $Q^2$ has the form in $QCD$ [1]:

$$\frac{d\sigma_L^{\gamma^* p \to V p}}{dt}\bigg|_{t=0} = \frac{12\pi^3 \Gamma_{V \to e^+ e^-} m_V \alpha_s^2(Q_1) Q^2 \eta_V^2 F^2(Q^2)}{\alpha_{EM}(Q^2 + 4m_c^2)^4 N_c^2} \left| \left(1 + i\frac{\pi}{2}\frac{d}{d\ln x}\right) xG(x, Q_1^2) \right|^2 . \quad (1)$$

Here, $-Q^2$ is the "$mass$"$^2$ of the virtual photon , $m_c$ is the current mass of the produced heavy quark $c$ and $\Gamma_{V \to e^+ e^-}$ is the decay width of the vector meson into a $e^+ e^-$ pair. $\eta_V = \frac{\int \frac{dz}{2z(1-z)}\psi_V(z)}{\int dz \psi_V(z)}$ and $\psi_V(z)$ is the minimal Fock component of the light–cone wave function of the vector meson with mass $m_V$. $F(Q^2)$ is a $Q^2$ dependent function to be discussed below. We want to stress that cross sections of hard diffractive processes as well as any hard processes can be simply expressed through the distribution of bare quarks and gluons but not through the distribution of constituent quarks. The factor $xG(x, Q_1^2)$ is the gluon distribution in the proton target. Within the leading ln approximations there is ambiguity in the dependence of parton distributions on the virtuality $Q^2$. To draw attention to this important point we introduce the variable $Q_1^2$. M. Ryskin suggested [2] to use $Q_1^2 = \frac{Q^2 + 4m_c^2}{2}$. Another intuitive suggestion is to compare parton distributions for the different processes at the same transverse distances between quarks in the wave function of the photon [3]. For moderate $Q^2$ the second suggestion predicts a somewhat less steep increase of the cross section with decreasing $x$. In principle due to the difference between the photon "mass" and the mass of the vector meson, nondiagonal gluon distributions should enter into the formulae [4]. But at sufficiently small $x$ it should coincide with the diagonal gluon distributions. This has been proven by the direct calculations within the leading $\alpha_s \ln x \ln Q^2$ approximation in [1]. A more general proof will be given elsewhere. By definition, the factor $F(Q^2)$ includes nonasymptotic effects related to the photon and the vector meson wave functions.

When $t$ is different from 0, the vertex for the transition $\gamma^* \to V$ should depend rather weakly on the momentum transferred to the target and should be almost universal for all hard

diffractive processes [4, 3]. If we denote by $B$ the slope of the differential cross section $d\sigma/dt$, we may estimate its value at large $Q^2$ as $B \approx \frac{R^2}{3}$. Here R is the radius of the gluon distribution in the target proton. Taking R=0.6fm from the realistic quark-gluon models of a nucleon we obtain $B = 3\text{GeV}^{-2}$. At $Q^2 = 0$ the photon-$J/\psi$ vertex leads to an additional contribution to $B$ of $\approx 0.7$ GeV$^2$ [3]. Thus a natural estimate for the slope of hard diffractive photoproduction of $J/\psi$ is $B(Q^2 = 0) = 4$ GeV$^{-2}$. With increasing $Q^2$ the slope B should decrease to the value around 3 GeV$^2$.

# 2    Comparison between different approximations

The cross section of diffractive photo - and electroproduction of $J/\Psi$ mesons has been evaluated by Ryskin [2] in terms of the BFKL approximation and nonrelativistic constituent quark charmonium model for the $J/\Psi$ meson. Brodsky et al. [1] used the leading $\alpha_s \ln \frac{Q^2}{\Lambda^2_{QCD}} \ln \frac{1}{x}$ approximation to evaluate the cross section of the electroproduction of longitudinally polarized vector mesons in terms of the minimal Fock component of the light-cone wave function of vector mesons. In [3] this process has been evaluated within the leading $\alpha_s \ln \frac{Q^2}{\Lambda^2_{QCD}}$ approximation of $QCD$ and effects of the quark Fermi motion were taken into account. In this case conventional leading order gluon and quark distributions enter, and not asymptotical gluon distributions as in [2] and/or in [1].

Within the approximation which assumes that the minimal Fock component of the light-cone wave function of the $J/\psi$ can be approximated by the wave function of a nonrelativistic charmonium model [2] the production of transversely polarized $J/\psi$ can be recalculated through the cross section of longitudinally polarized $J/\psi$. If the real part of the amplitude as well as the quark Fermi motion effects are neglected the calculation of [1] leads at small $Q^2$ to the formulae of [2] [1]. At the same time at large $Q^2$ the wave functions of all mesons should approach the universal wave function $z(1 - z)$ [5] and the charmonium model approximation would be in variance with $QCD$. (Here $z$ is the fraction of the momentum of the vector meson carried by the quark).

The effects of the quark Fermi motion within this model are given by the factor $F^2(Q^2)$,

$$F(Q^2) = \frac{\int d^3k \left(\frac{1}{m_c^2+k^2}\right)^{\frac{1}{4}} \psi_V(k) \frac{-\Delta}{4} \frac{1}{\left(\frac{Q^2}{4}+(m_c^2+k^2)\right)}}{\int d^3k \frac{\psi_V(k)\left(\frac{1}{m_c^2+k^2}\right)^{\frac{1}{4}}}{\left(\frac{Q^2}{4}+m_c^2\right)^2}} \,, \tag{2}$$

The substitution $z = 0.5(1 + \frac{k_3}{\sqrt{k^2+m_c^2}})$ is necessary to transform diagrams of light cone perturbation theory for the wave function of $J/\psi$ into nonrelativistic diagrams for the wave function of $J/\psi$ in charmonium models. The factor $(k^2 + m_c^2)^{-1/4}$ is included in the definition of the

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[1]In Ref. [1], a factor 4 was missed in the numerator of Eq. (1). We are indebted to Z. Chan for pointing this out.

nonrelativistic charmonium wave function to keep the phase volume to be the same as within the nonrelativistic approach. Here $\Delta$ is the Laplace operator in transverse momentum space which acts on the photon wave function.

In [3] realistic charmonium wave functions calculated from a power-law [6] and a logarithmic potential [7] were used. Both functions describe $\Gamma_{J/\Psi \rightarrow e^+ e^-}$ reasonably well. The evaluation leads to the factor $F^2(0) \approx 1/9$. The significant suppression of the cross section at moderate $Q^2$ is caused by the fact that for the realistic charmonium model wave functions the integral over the quark momenta is slowly convergent at large quark momenta .

To visualize the estimate of $F(Q^2 = 0)$ we decompose it into powers of $\frac{<k^2>}{m_c^2}$: $F^2(0) = 1 - c < \frac{k^2}{m_c^2} >$. Here by definition $< k^2 > = \frac{\int k^2 \psi_V(k) d^3 k}{\int \psi_V(k) d^3 k}$. (Note that for the Coulomb potential- which follows from QCD for the sufficiently large mass of heavy flavor $\int k^2 \psi_V(k) d^3 k = \infty$. So this decomposition is inapplicable for the production of sufficiently heavy flavors where applicability of $pQCD$ is most reliable.) The coefficient $c$ has been evaluated in [8] as $c = 4$ and in [3] as $c = 6\frac{2}{3}$ . This difference is due to the leading and next to leading powers of $\frac{k^2}{m_c^2}$ in the evaluation of the operator $\Delta$ both of which are taken into account in [3] but not in [8]. For the numerical estimates Ryskin et al. [8] used a gaussian type approximation to the wave function of the $J/\psi$ and suggested $< k^2/m_c^2 > \approx 1/8$. Within this approximation $F^2(0) \approx 0.5$ for $c = 4$ [8] and $F^2(0) \approx \frac{1}{6}$ for $c = 6\frac{2}{3}$ [3] . The numerical approximations leading to the difference between the calculations of [3] and of [8] are discussed in detail in [3]. This problem deserves further investigation.

If the dependence of the interquark potential on the mass of the quark (on the running coupling constant, on the running quark mass etc ) can be neglected, the cross section would be $\sigma \approx \frac{1}{m_c^6}$. Within the range of constituent quark masses between $m_c = 1.48 - 1.54$ GeV which are explored in the realistic charmonium models, and at the same time not far from the bare mass of the $c$ quark the cross section changes within 42%. Calculations within the realistic charmonium models produce weaker dependence on $m_c$ [3].

# 3   Restoration of flavor symmetry

For the production of vector mesons with $M_V^2 \ll Q^2$ by longitudinally polarized photons, all dependence on the quark masses and thus on flavor is contained in the light-cone wave functions of the vector mesons only. Color exchange is flavor blind and it is legitimate to neglect the masses of current quarks in the hard scattering amplitude . $SU(4)$ predicts that at sufficiently large $Q^2$ the ratio of the production cross section for charmed mesons to that of the $\rho^0$ is $J/\Psi : \rho^0 = 8 : 9$ . The production of $J/\Psi$ should be additionally enhanced at large $Q^2$ by the larger probability for a smaller object to annihilate into small object. Experimentally, this ratio is suppressed at small $Q^2$ as compared to the prediction of $SU(4)$ symmetry by a factor of $\approx 25$. Such a suppression is naturally explained within the above discussed model and it should slowly disappear with increasing $Q^2$ [3]. Thus QCD predicts [2] a dramatic increase of the $J/\Psi/\rho^o$ ratios at large $Q^2$ [4].

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[2]Note however that this view is not shared by M.Ryskin.

At very large $Q^2 \gg M_V^2$, the $q\bar{q}$ wave functions of all mesons should converge to a universal asymptotic wave function [5]. In this limit, further enhancement of heavy flavor production is expected leading to $J/\Psi : \rho^0 = (8 * 3.4) : 9$ [4]. The analysis of the factor $F(Q^2)$ discussed above shows that the observation of the full restoration of the predictions of $SU(4)$ symmetry for the $J/\Psi$–meson production would require extremly high values of $Q^2$.

# 4 Comparison with the HERA data

The presently published results of the HERA experiments on diffractively produced $J/\Psi$ is for $Q^2 = 0$ [9, 10]. Some preliminary results in the electroproduction region ($< Q^2 >= 18$ GeV$^2$) have been presented by H1 at the EPS meeting in Brussels [11].
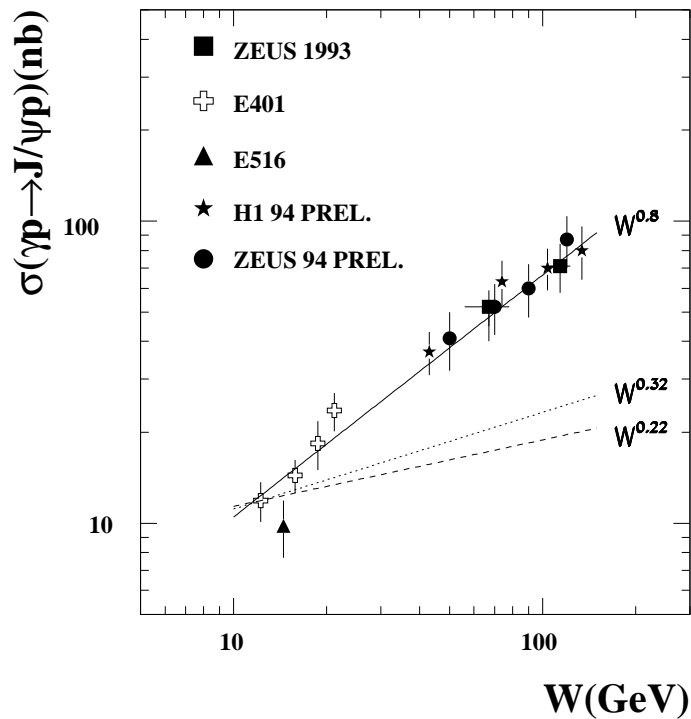


Figure 1: A compilation of $J/\Psi$ elastic cross sections in photoproduction. The solid line is a $W^{0.8}$ behaviour while the dotted and dashed lines are the predictions following the Donnachie and Landshoff model[12] without ($W^{0.32}$) and with ($W^{0.22}$) shrinkage.

A compilation of the photoproduction cross section for the reaction $\gamma p \to J/\Psi p$ as function of the $\gamma p$ center of mass energy $W$ is presented in figure 1. One can see a fast increase of the cross section with $W$, approximately like $W^{0.8-0.9}$. Note that this increase is also seen in the HERA data alone, thus independent of possible relative normalizations between the low and high energy experiments. Such a fast increase of the cross section with $W$ is unexpected within the non–perturbative two–gluon exchange of Donnachie and Landshoff [12], which predict a rise

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like $W^{0.22}$ after taking shrinkage into account. The rise is consistent with a model calculated within leading $\ln x$ approximation which connects the rise to the behaviour of the gluon density with decreasing $x$ [2].

The preliminary data from the electroproduction of $J/\Psi$ [11] indicate that the $Q^2$ dependence is steeper than that expected from GVDM. However there are two QCD effects, which tend to slow down the dependence on $Q^2$. One effect is that the gluon density increases with $Q^2$ at small $x$. Numerically, the factor $\alpha_s^2(Q^2)x^2G_N^2(x, Q^2)$ in Eq. (1) is $\propto Q^n$ with $n \sim 1$. In addition a *preasymptotic* effect comes from the factor $F(Q^2)$ of Eq. (2). The combination of these two effects results in a relatively slow decrease with $Q^2$ as compared to $\frac{1}{(Q^2+4m_c^2)^3}$. This could be checked once higher statistics data are available.

# 5    Summary

To summarize, the investigation of exclusive diffractive processes appears to be an effective method to measure the minimal Fock state $q\bar{q}$ component of the light-cone wave functions of vector mesons as well as the light-cone wave functions of any small mass hadron system having angular momentum one.

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